<http://www.physics.csbsju.edu/cgi-bin/stats/anova_pnp>

**ANOVA: Results**

The results of a ANOVA statistical test performed at 20:24 on 8-JAN-2018

Source of Sum of d.f. Mean F

Variation Squares Squares

between 2.2901E+05 5 4.5802E+04 3.359

error 8.7820E+06 644 1.3637E+04

total 9.0110E+06 649

The probability of this result, assuming the null hypothesis, is 0.0053

Group A: Number of items=104  
5.00 10.0 12.0 14.0 14.0 15.0 17.0 17.0 18.0 19.0 20.0 22.0 24.0 28.0 29.0 33.0 34.0 34.0 35.0 35.0 37.0 38.0 39.0 43.0 43.0 44.0 45.0 45.0 49.0 51.0 53.0 55.0 56.0 58.0 58.0 62.0 63.0 66.0 68.0 69.0 69.0 70.0 76.0 77.0 77.0 79.0 80.0 81.0 82.0 83.0 84.0 84.0 86.0 88.0 94.0 97.0 99.0 101. 102. 104. 104. 105. 105. 110. 114. 115. 119. 121. 124. 133. 133. 135. 147. 148. 153. 161. 164. 173. 174. 174. 178. 180. 181. 184. 203. 205. 205. 210. 218. 218. 237. 246. 254. 257. 262. 268. 405. 410. 499. 595. 604. 667. 693. 698.

Mean = 134.41   
95% confidence interval for Mean: 111.9 thru 156.9   
Standard Deviation = 147.   
High = 698.0 Low = 5.000   
Median = 85.00   
Average Absolute Deviation from Median = 87.9

Group B: Number of items=104  
1.00 1.00 1.00 2.00 3.00 5.00 7.00 12.0 12.0 13.0 21.0 22.0 22.0 23.0 26.0 29.0 29.0 30.0 31.0 32.0 32.0 34.0 35.0 37.0 39.0 40.0 42.0 42.0 43.0 44.0 47.0 47.0 48.0 49.0 50.0 51.0 55.0 56.0 56.0 57.0 59.0 61.0 64.0 65.0 65.0 65.0 68.0 70.0 72.0 77.0 79.0 79.0 80.0 80.0 82.0 82.0 82.0 82.0 86.0 87.0 88.0 98.0 98.0 98.0 99.0 101. 102. 106. 107. 108. 109. 110. 111. 113. 117. 120. 121. 122. 124. 124. 125. 133. 137. 143. 150. 162. 178. 182. 207. 210. 229. 230. 231. 237. 268. 271. 277. 284. 331. 352. 387. 392. 477. 665.

Mean = 107.16   
95% confidence interval for Mean: 84.68 thru 129.6   
Standard Deviation = 108.   
High = 665.0 Low = 1.000   
Median = 79.50   
Average Absolute Deviation from Median = 67.7

Group C: Number of items= 93  
10.0 12.0 13.0 15.0 18.0 18.0 19.0 19.0 23.0 23.0 25.0 26.0 26.0 29.0 31.0 32.0 38.0 41.0 42.0 43.0 45.0 46.0 47.0 49.0 51.0 53.0 56.0 58.0 60.0 63.0 69.0 70.0 70.0 71.0 73.0 76.0 77.0 77.0 78.0 78.0 82.0 84.0 84.0 84.0 84.0 88.0 90.0 96.0 98.0 101. 101. 102. 106. 108. 109. 112. 115. 117. 120. 124. 127. 129. 129. 129. 133. 134. 144. 145. 147. 158. 161. 168. 186. 194. 200. 205. 209. 221. 228. 230. 236. 237. 256. 272. 359. 361. 379. 390. 541. 564. 627. 904. 947.

Mean = 142.20   
95% confidence interval for Mean: 118.4 thru 166.0   
Standard Deviation = 166.   
High = 947.0 Low = 10.00   
Median = 90.00   
Average Absolute Deviation from Median = 92.3

Group D: Number of items=130  
7.00 8.00 9.00 9.00 17.0 21.0 21.0 22.0 22.0 23.0 24.0 24.0 25.0 25.0 26.0 27.0 28.0 29.0 32.0 32.0 32.0 32.0 32.0 33.0 33.0 36.0 36.0 36.0 37.0 39.0 39.0 41.0 43.0 43.0 44.0 46.0 47.0 50.0 50.0 50.0 51.0 52.0 52.0 52.0 54.0 57.0 58.0 59.0 61.0 62.0 64.0 65.0 65.0 68.0 74.0 77.0 80.0 80.0 81.0 82.0 83.0 84.0 84.0 85.0 85.0 86.0 87.0 89.0 89.0 89.0 90.0 91.0 92.0 92.0 94.0 96.0 96.0 97.0 99.0 102. 104. 105. 106. 109. 110. 111. 114. 115. 117. 118. 118. 120. 122. 127. 128. 138. 139. 141. 142. 144. 147. 147. 149. 149. 151. 152. 153. 156. 158. 159. 159. 168. 177. 178. 185. 201. 207. 219. 223. 244. 247. 260. 265. 274. 284. 301. 307. 317. 377. 437.

Mean = 102.41   
95% confidence interval for Mean: 82.30 thru 122.5   
Standard Deviation = 79.9   
High = 437.0 Low = 7.000   
Median = 85.50   
Average Absolute Deviation from Median = 57.1

Group E: Number of items=114  
18.0 20.0 31.0 33.0 36.0 37.0 38.0 38.0 39.0 41.0 41.0 42.0 49.0 49.0 49.0 50.0 53.0 54.0 56.0 59.0 59.0 62.0 63.0 64.0 67.0 67.0 72.0 74.0 75.0 78.0 78.0 81.0 83.0 83.0 83.0 83.0 83.0 84.0 85.0 86.0 90.0 92.0 94.0 94.0 97.0 98.0 98.0 102. 105. 105. 109. 110. 110. 111. 112. 116. 122. 125. 127. 128. 128. 129. 129. 130. 130. 132. 133. 135. 136. 138. 139. 140. 140. 142. 143. 147. 156. 159. 159. 162. 162. 163. 167. 170. 174. 175. 182. 191. 194. 194. 203. 210. 212. 213. 218. 224. 224. 226. 245. 254. 264. 267. 273. 275. 285. 285. 302. 318. 326. 346. 370. 383. 407. 414.

Mean = 138.69   
95% confidence interval for Mean: 117.2 thru 160.2   
Standard Deviation = 89.0   
High = 414.0 Low = 18.00   
Median = 123.5   
Average Absolute Deviation from Median = 66.6

Group F: Number of items=105  
14.0 25.0 30.0 42.0 45.0 49.0 52.0 52.0 52.0 55.0 55.0 56.0 59.0 60.0 63.0 63.0 64.0 65.0 66.0 67.0 68.0 68.0 68.0 69.0 71.0 71.0 72.0 74.0 74.0 79.0 79.0 81.0 83.0 83.0 83.0 86.0 90.0 94.0 102. 102. 103. 106. 107. 111. 111. 116. 117. 117. 121. 126. 130. 133. 139. 141. 141. 143. 143. 144. 145. 146. 150. 151. 155. 156. 159. 162. 166. 167. 168. 169. 178. 179. 182. 182. 184. 189. 193. 194. 196. 198. 199. 200. 202. 205. 207. 211. 216. 218. 225. 251. 251. 258. 260. 262. 263. 282. 302. 349. 351. 359. 364. 384. 430. 499. 610.

Mean = 152.45   
95% confidence interval for Mean: 130.1 thru 174.8   
Standard Deviation = 103.   
High = 610.0 Low = 14.00   
Median = 139.0   
Average Absolute Deviation from Median = 74.4

http://astatsa.com/OneWay\_Anova\_with\_TukeyHSD/\_result/

Para los dos casos de estudio por separado:

#### Conclusion from Anova:

The p-value corresponing to the F-statistic of one-way ANOVA is lower than 0.05, suggesting that the one or more treatments are significantly different. The Tukey HSD test, Scheffé, Bonferroni and Holm multiple comparison tests follow. These post-hoc tests would likely identify which of the pairs of treatments are significantly differerent from each other.

Being D = PB and F=PC

### *post-hoc Tukey HSD Test Calculator results:*

k=6k=6 treatments   
degrees of freedom for the error term ν=644ν=644   
Critical values of the Studentized Range QQ statistic:

Qα=0.01,k=6,ν=644criticalQcriticalα=0.01,k=6,ν=644 = 4.7781               Qα=0.05,k=6,ν=644criticalQcriticalα=0.05,k=6,ν=644 = 4.0422

We present below color coded results (red for insignificant, green for significant) of evaluating whether Qi,j>QcriticalQi,j>Qcritical for all relevant pairs of treatments. In addition, we also present the significance (p-value) of the observed QQ-statistic Qi,jQi,j. The algorithm used here to calculate the critical values of the studentized range distribution, as well as p-values corresponding to an observed value of Qi,jQi,j, is that of [Gleason (1999)](http://www.sciencedirect.com/science/article/pii/S016794739900002X). This is an improvement over the [Copenhaver-Holland (1988) algorithm](http://www.tandfonline.com/doi/abs/10.1080/00949658808811082) deployed in [the R statistical package](http://stat.ethz.ch/R-manual/R-patched/library/stats/html/Tukey.html).

#### Tukey HSD results

| treatments  pair | Tukey HSD  Q statistic | Tukey HSD  p-value | Tukey HSD  inferfence |
| --- | --- | --- | --- |
| A vs B | 2.3797 | 0.5375261 | insignificant |
| A vs C | 0.6611 | 0.8999947 | insignificant |
| A vs D | 2.9462 | 0.2973791 | insignificant |
| A vs E | 0.3822 | 0.8999947 | insignificant |
| A vs F | 1.5787 | 0.8638807 | insignificant |
| B vs C | 2.9734 | 0.2872624 | insignificant |
| B vs D | 0.4378 | 0.8999947 | insignificant |
| B vs E | 2.8159 | 0.3488486 | insignificant |
| B vs F | 3.9641 | 0.0583028 | insignificant |
| C vs D | 3.5487 | 0.1231304 | insignificant |
| C vs E | 0.3043 | 0.8999947 | insignificant |
| C vs F | 0.8712 | 0.8999947 | insignificant |
| D vs E | 3.4247 | 0.1502242 | insignificant |
| D vs F | 4.6186 | 0.0145535 | \* p<0.05 |
| E vs F | 1.2315 | 0.8999947 | insignificant |

### *Scheffé multiple comparison*

We define a statistic named TT as the ratio of unsigned contrast mean to contrast standard error, as explained in the [NIST Engineering Statistics Handbook page for Scheffe's method](http://www.itl.nist.gov/div898/handbook/prc/section4/prc472.htm). It can be show that for contrasts that are treatment pairs (i,j)(i,j) with unit coefficents,

Ti,j=Qi,j2–√Ti,j=Qi,j2

where Qi,jQi,j is the QQ-statistic that was created for the Tukey HSD test. This TT-statistic has interesting properties.

The same [NIST Engineering Statistics Handbook page for Scheffe's method](http://www.itl.nist.gov/div898/handbook/prc/section4/prc472.htm)provides a formula which directly leads to the Scheffé p-value corresponding to an observed value of TT as:

1−F(T2k−1,k−1,ν)1−F(T2k−1,k−1,ν)

where F()F() is the cumulative FF distribution with its two degrees of freedom parameters k−1k−1 and νν. Note that kkis the number of treatments and νν is the degrees of freedom of error that were established earlier.

The Scheffé p-value of the observed TT-statistic Ti,jTi,j is shown below for all relevant pairs of treatments, along with color coded Scheffé inference (red for insignificant, green for significant) based on the p-value.

#### Scheffé results

|  |  |  |  |
| --- | --- | --- | --- |
| treatments  pair | Scheffé  TT-statistic | Scheffé  p-value | Scheffé  inferfence |
| A vs B | 1.6827 | 0.7258771 | insignificant |
| A vs C | 0.4675 | 0.9988956 | insignificant |
| A vs D | 2.0833 | 0.5021641 | insignificant |
| A vs E | 0.2703 | 0.9999249 | insignificant |
| A vs F | 1.1163 | 0.9402092 | insignificant |
| B vs C | 2.1025 | 0.4912230 | insignificant |
| B vs D | 0.3096 | 0.9998531 | insignificant |
| B vs E | 1.9911 | 0.5549414 | insignificant |
| B vs F | 2.8030 | 0.1659923 | insignificant |
| C vs D | 2.5093 | 0.2798680 | insignificant |
| C vs E | 0.2152 | 0.9999757 | insignificant |
| C vs F | 0.6160 | 0.9958564 | insignificant |
| D vs E | 2.4216 | 0.3209620 | insignificant |
| D vs F | 3.2658 | 0.0598468 | insignificant |
| E vs F | 0.8708 | 0.9795374 | insignificant |

### *Bonferroni and Holm multiple comparison*

The same statistic TT for the Scheffé method, along with the number of contrasts (pairs) qq being simultaneously compared, leads to the Bonferroni formula. The [NIST Engineering Statistics Handbook page for Bonferroni method](http://www.itl.nist.gov/div898/handbook/prc/section4/prc473.htm)provides a formula which directly leads to the Bonferroni p-value corresponding to an observed value of TT in the context of simultaneous comparison of qq contrasts as:   
Bonferroni p-value:     PBonferronii,j=Punadjustedi,jqPi,jBonferroni=Pi,junadjustedq     where

Punadjustedi,j=[1−t(T2k−1,ν)]2Pi,junadjusted=[1−t(T2k−1,ν)]2

and where t()t() is the cumulative Student's tt distribution with its degree of freedom parameter νν. Note that νν is the degrees of freedom of error that were established earlier. Also note that the p-value of Bonferroni simultaneous comparison is directly proportional to qq, the number of contrasts (pairs) being simultaneously compared.

The Holm procedure described in [Aickin and Gensler (1996) review paper](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1380484/)requires sorting the Punadjustedi,jPi,junadjusted as above in *ascending* order and determining the sort rank Ri,jRi,j of each unique pair (i,j)(i,j). These sort ranks run from 1 through qq. The Holm p-value for comparing a given pair (i,j)(i,j) in the context of multiple comparison of qq such pairs simultaneously is:   
  
Holm p-value:     PHolmi,j=Punadjustedi,j(q−Ri,j+1)Pi,jHolm=Pi,junadjusted(q−Ri,j+1)

In this first combined Bonferroni and Holm table below, we consider all possible contrasts (pairs) for simultaneous comparion, thus qq=15. The Bonferoni and Holm p-values of the observed TT-statistic Ti,jTi,j for all relevant qq=15 pairs of treatments is shown below, along with color coded Bonferroni and Holm inferences (red for insignificant, green for significant) based on the p-value.

#### Bonferroni and Holm results: all pairs simultaineously compared

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| treatments  pair | Bonferroni  and Holm  TT-statistic | Bonferroni  p-value | Bonferroni  inferfence | Holm  p-value | Holm  inferfence |
| A vs B | 1.6827 | 1.3936895 | insignificant | 0.7433011 | insignificant |
| A vs C | 0.4675 | 9.6048145 | insignificant | 2.5612839 | insignificant |
| A vs D | 2.0833 | 0.5642501 | insignificant | 0.3761667 | insignificant |
| A vs E | 0.2703 | 11.8057041 | insignificant | 1.5740939 | insignificant |
| A vs F | 1.1163 | 3.9706736 | insignificant | 1.8529810 | insignificant |
| B vs C | 2.1025 | 0.5383986 | insignificant | 0.3948256 | insignificant |
| B vs D | 0.3096 | 11.3549205 | insignificant | 2.2709841 | insignificant |
| B vs E | 1.9911 | 0.7032987 | insignificant | 0.4219792 | insignificant |
| B vs F | 2.8030 | 0.0782194 | insignificant | 0.0730048 | insignificant |
| C vs D | 2.5093 | 0.1851225 | insignificant | 0.1604395 | insignificant |
| C vs E | 0.2152 | 12.4452958 | insignificant | 0.8296864 | insignificant |
| C vs F | 0.6160 | 8.0715564 | insignificant | 2.6905188 | insignificant |
| D vs E | 2.4216 | 0.2359029 | insignificant | 0.1887223 | insignificant |
| D vs F | 3.2658 | 0.0172394 | \* p<0.05 | 0.0172394 | \* p<0.05 |
| E vs F | 0.8708 | 5.7627974 | insignificant | 2.3051190 | insignificant |

In this second Bonferroni and Holm table below, we consider a subset of contrasts (pairs) for simultaneous comparion, of only pairs relative to treatment A. Such a situation may be relevant when treatment A is the control, and the experimenter is interested only in differences of treatments relative to control, thus qq=5. The Bonferoni and Holm p-values of the observed TT-statistic Ti,jTi,j for qq=5 relevant pairs of treatments, along with color coded Bonferroni inference (red for insignificant, green for significant) based on the p-value.

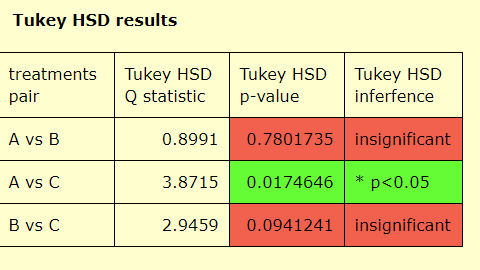
#### Bonferroni and Holm results: only pairs relative to A simultaineously compared

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| treatments  pair | Bonferroni  and Holm  TT-statistic | Bonferroni  p-value | Bonferroni  inferfence | Holm  p-value | Holm  inferfence |
| A vs B | 1.6827 | 0.4645632 | insignificant | 0.3716505 | insignificant |
| A vs C | 0.4675 | 3.2016048 | insignificant | 1.2806419 | insignificant |
| A vs D | 2.0833 | 0.1880834 | insignificant | 0.1880834 | insignificant |
| A vs E | 0.2703 | 3.9352347 | insignificant | 0.7870469 | insignificant |
| A vs F | 1.1163 | 1.3235579 | insignificant | 0.7941347 | insignificant |

Para B, BC and C.

#### Conclusion from Anova:

The p-value corresponing to the F-statistic of one-way ANOVA is lower than 0.05, suggesting that the one or more treatments are significantly different. The Tukey HSD test, Scheffé, Bonferroni and Holm multiple comparison tests follow. These post-hoc tests would likely identify which of the pairs of treatments are significantly differerent from each other.



### *Scheffé multiple comparison*

We define a statistic named TT as the ratio of unsigned contrast mean to contrast standard error, as explained in the [NIST Engineering Statistics Handbook page for Scheffe's method](http://www.itl.nist.gov/div898/handbook/prc/section4/prc472.htm). It can be show that for contrasts that are treatment pairs (i,j)(i,j) with unit coefficents,

Ti,j=Qi,j2–√Ti,j=Qi,j2

where Qi,jQi,j is the QQ-statistic that was created for the Tukey HSD test. This TT-statistic has interesting properties.

The same [NIST Engineering Statistics Handbook page for Scheffe's method](http://www.itl.nist.gov/div898/handbook/prc/section4/prc472.htm)provides a formula which directly leads to the Scheffé p-value corresponding to an observed value of TT as:

1−F(T2k−1,k−1,ν)1−F(T2k−1,k−1,ν)

where F()F() is the cumulative FF distribution with its two degrees of freedom parameters k−1k−1 and νν. Note that kkis the number of treatments and νν is the degrees of freedom of error that were established earlier.

The Scheffé p-value of the observed TT-statistic Ti,jTi,j is shown below for all relevant pairs of treatments, along with color coded Scheffé inference (red for insignificant, green for significant) based on the p-value.

#### Scheffé results

|  |  |  |  |
| --- | --- | --- | --- |
| treatments  pair | Scheffé  TT-statistic | Scheffé  p-value | Scheffé  inferfence |
| A vs B | 0.6357 | 0.8170771 | insignificant |
| A vs C | 2.7375 | 0.0241002 | \* p<0.05 |
| B vs C | 2.0831 | 0.1150594 | insignificant |

### *Bonferroni and Holm multiple comparison*

The same statistic TT for the Scheffé method, along with the number of contrasts (pairs) qq being simultaneously compared, leads to the Bonferroni formula. The [NIST Engineering Statistics Handbook page for Bonferroni method](http://www.itl.nist.gov/div898/handbook/prc/section4/prc473.htm)provides a formula which directly leads to the Bonferroni p-value corresponding to an observed value of TT in the context of simultaneous comparison of qq contrasts as:   
Bonferroni p-value:     PBonferronii,j=Punadjustedi,jqPi,jBonferroni=Pi,junadjustedq     where

Punadjustedi,j=[1−t(T2k−1,ν)]2Pi,junadjusted=[1−t(T2k−1,ν)]2

and where t()t() is the cumulative Student's tt distribution with its degree of freedom parameter νν. Note that νν is the degrees of freedom of error that were established earlier. Also note that the p-value of Bonferroni simultaneous comparison is directly proportional to qq, the number of contrasts (pairs) being simultaneously compared.

The Holm procedure described in [Aickin and Gensler (1996) review paper](http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1380484/)requires sorting the Punadjustedi,jPi,junadjusted as above in *ascending* order and determining the sort rank Ri,jRi,j of each unique pair (i,j)(i,j). These sort ranks run from 1 through qq. The Holm p-value for comparing a given pair (i,j)(i,j) in the context of multiple comparison of qq such pairs simultaneously is:   
  
Holm p-value:     PHolmi,j=Punadjustedi,j(q−Ri,j+1)Pi,jHolm=Pi,junadjusted(q−Ri,j+1)

In this first combined Bonferroni and Holm table below, we consider all possible contrasts (pairs) for simultaneous comparion, thus qq=3. The Bonferoni and Holm p-values of the observed TT-statistic Ti,jTi,j for all relevant qq=3 pairs of treatments is shown below, along with color coded Bonferroni and Holm inferences (red for insignificant, green for significant) based on the p-value.

#### Bonferroni and Holm results: all pairs simultaineously compared

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| treatments  pair | Bonferroni  and Holm  TT-statistic | Bonferroni  p-value | Bonferroni  inferfence | Holm  p-value | Holm  inferfence |
| A vs B | 0.6357 | 1.5755053 | insignificant | 0.5251684 | insignificant |
| A vs C | 2.7375 | 0.0190805 | \* p<0.05 | 0.0190805 | \* p<0.05 |
| B vs C | 2.0831 | 0.1129162 | insignificant | 0.0752775 | insignificant |

In this second Bonferroni and Holm table below, we consider a subset of contrasts (pairs) for simultaneous comparion, of only pairs relative to treatment A. Such a situation may be relevant when treatment A is the control, and the experimenter is interested only in differences of treatments relative to control, thus qq=2. The Bonferoni and Holm p-values of the observed TT-statistic Ti,jTi,j for qq=2 relevant pairs of treatments, along with color coded Bonferroni inference (red for insignificant, green for significant) based on the p-value.

#### Bonferroni and Holm results: only pairs relative to A simultaineously compared

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| treatments  pair | Bonferroni  and Holm  TT-statistic | Bonferroni  p-value | Bonferroni  inferfence | Holm  p-value | Holm  inferfence |
| A vs B | 0.6357 | 1.0503368 | insignificant | 0.5251684 | insignificant |
| A vs C | 2.7375 | 0.0127203 | \* p<0.05 | 0.0127203 | \* p<0.05 |

### H